11. Conservation of Energy

Objectives

A cart is pushed and released to travel up and down an inclined track.

* Determine the relationship between the kinetic, potential, and total mechanical energies.

Materials and Equipment

For each student or group:

* LabQuest Data collection system  
* Motion Detector  
* Dynamics Track  
* Dynamics cart  
* 2 books  
* Meter Stick

Background

An object in the presence of a net conservative force experiences acceleration related to Newton's second law. As the object's velocity increases due to this acceleration its kinetic energy $K$ increases in the form:

$$K = \frac{1}{2}mv^2$$  \hspace{1cm} (1)

Where $m$ is the mass of the object and $v$ is the object's instantaneous velocity.

Imagine that this object was tossed up into the air and rose to some height $h$ above the ground and returned back down to where it was released. At the beginning of its motion through the air, the object would have some kinetic energy. As it rose, it would slow, losing kinetic energy in the process. At the top of the object's motion, the kinetic energy would be zero because its velocity is zero. And as it falls back down, its kinetic energy would increase back to its original value. The kinetic energy of the object is clearly not conserved.

Where does the kinetic energy go, and how does it reappear? There is another form of energy involved in the rising and falling of an object and that is gravitational potential energy $U$:

$$U = PE_g = mgh$$  \hspace{1cm} (2)

Potential energy is described as the energy stored in a system that has the potential to be converted into kinetic energy. When the object was released at its lowest point, it had no gravitational potential energy. All of its energy was in the form of kinetic energy. As it rose into the air, kinetic energy of motion was converted into potential energy of position. When it reached a height $h$ it had no kinetic energy because its velocity was zero; however, the object did have potential energy equal to $mgh$.

Because the only force acting on the object is a conservative force the energy stored in the object must also be conservative, which implies that the amount of kinetic energy the object loses as it rises must equal the amount of potential energy that it has gained. In other words, the object's kinetic and potential energies may change as height and velocity change, but the total energy in...
the system at any height \( h \) and velocity \( v \) stays constant. This constant energy is known as mechanical energy \( E \):

\[
E = U + K = mgh + \frac{1}{2}mv^2 = \text{constant}
\]  

(3)

In the case of a cart on a track, Eq. 3 holds true for any height \( h \) above the lowest part on the track and any corresponding velocity \( v \), even though the motion of the cart may not be in the direction of the conservative force being applied. This is true because the applied force is still responsible for the motion of the cart.

According to Eq. 3 the energy equation representing the cart on the curved track in Figure 1 would look like:

\[
E_A = E_B = E_C = E_D = \ldots \quad \text{or} \quad mgh_A + \frac{1}{2}mv_A^2 = mgh_B + \frac{1}{2}mv_B^2 = mgh_C + \frac{1}{2}mv_C^2 = \ldots
\]

**Relevant Equations**

\[
K = \frac{1}{2}mv^2
\]

\[
U = mgh
\]

\[
E = U + K = mgh + \frac{1}{2}mv^2 = \text{constant}
\]

**Safety**

Add this important safety precaution to your normal laboratory procedures:

- The cart has the ability to roll off the end of the track and possibly knock objects off the lab bench. Make certain a student stops the cart before it rolls off the end of the track.
Procedure

After you complete a step (or answer a question), place a check mark in the box (☑) next to that step.

Set Up

1. ☐ Adjust the legs on the 2.2m track so that they are approximately 1.5 m apart. Place 2 text books under the legs at one end of the track. Attach stops to both ends of the track to prevent the cart from rolling off.

2. ☐ Attach the motion detector to the lower end of the track and aim the detector up the track. To accommodate the clamp, the end of the track must extend beyond the table end. Be careful to keep the legs on the table.

3. ☐ Connect the motion detector to the DIG 1 port on the LabQuest interface and connect the LabQuest to a laptop using the USB cable.


Collect Data

5. ☐ Measure the separation of the legs of the track to the nearest mm using the markings on the top of the track and record below.

   Separation of track legs: \( \sim 1.700 \text{ m} \)

6. ☐ Measure the height of the books under one set of legs to the nearest mm with a meter stick and record below.

   Height of books: \( \sim 0.060 \text{ m} \)

7. ☐ Practice giving the cart a quick push up the inclined track so that it moves close to, but not touching the top bumper.

8. ☐ Start data recording and give the cart a quick push up the track, allowing it to roll back down to the point where it was released. Try and stop the data recording when the cart reaches the 0.5 m mark from the detector. Catch the cart before it crashes.

9. ☐ The screen should display the position v. time, velocity v. time, and acceleration v. time kinematics graphs for the cart. You may repeat the previous step until you are satisfied with the shapes of those graphs.

10. ☐ Each member of your group should collect his/her own data.
Data Analysis

1. Use the value for the height of the book and the distance between the legs of the track to calculate the angle $\theta$ of the incline of the track. Show your work below:

   $d \approx 1.700\text{m}$
   $h \approx 0.060\text{m}$

   $\theta = \sin^{-1}\left(\frac{h}{d}\right) = \sin^{-1}\left(\frac{0.060\text{m}}{1.700\text{m}}\right) = 2.0^\circ$

   Actual values will vary.

2a. Using the Velocity v. Time graph, sketch your prediction of the Kinetic Energy v. Time graph below. Keep in mind that the kinetic energy $K = \frac{1}{2}mv^2$, where $m$ is the mass of the cart and $v$ is its velocity.

2b. Use the Next Page command in the menu bar to view Page 2 (KE) of the file. On this page you will see the Velocity v. Time graph at the top. On the Kinetic Energy v. Time graph at the bottom, click the vertical axis and select Kinetic Energy. Sketch the Kinetic Energy v. Time graph in the space below. Where is the KE zero? Where is it a maximum? Is KE conserved?

KE is zero when the cart is at maximum displacement.
KE is a maximum at the beginning and end of its motion.
KE is not conserved.
3a. Go back to page 1 and use the Position v. Time graph to sketch your prediction of the Potential Energy v. Time graph below. Keep in mind that the potential energy $U = mgh$, where $m$ is the mass of the cart and $h$ is its height above the starting point, and $g$ is the free-fall acceleration constant.

3b. Use the Next Page command in the menu bar to view Page 3 (PE) of the file. On this page you will see the Position v. Time graph at the top. On the Potential Energy v. Time graph at the bottom, click the vertical axis and select Potential Energy. Sketch the Potential Energy v. Time graph in the space below. Where is the PE zero? Where is it a maximum? Is PE conserved?

![Potential Energy Graph]

PE is zero at the beginning and end of the motion. PE is maximum when cart is at its maximum displacement. PE is not conserved.

4. The height $h$ in the equation for Potential Energy ($U = mgh$) can be found using the equation $h = (x - x_0)\sin \theta$, where $x$ is the distance from the detector, $x_0$ is the initial position, and $\theta$ is the angle of the incline. Calculate $h$ using position data from the Position v. Time graph and $\theta$ from your calculation in Q1.

$$h = (x - x_0)\sin \theta = 1.2 \text{ m} \sin (2.0^\circ) = 0.042 \text{ m}$$

Answers will vary

5a. Sketch below your prediction of the mechanical energy (the sum of kinetic and potential energies) of the cart over time as it moves.
5b. Use the Next Page command in the menu bar to view Page 4 (ME) of the file. On this page you will see the Kinetic Energy v. Time and Potential Energy v. Time graphs at the top of the page. On the bottom graph of Mechanical Energy v. Time, click the vertical axis and select “More” so you can display Kinetic Energy, Potential Energy, and Mechanical Energy. Sketch this graph in the space below.

6. Print the graph that you created in 5b by selecting Print Graph from the File menu while the Mechanical Energy graph is selected. Be sure to format it in landscape. Attach the graph to this lab.

Analysis Questions

1. Describe all energy transformations that occur as the cart moves along the track. Include the initial push and the final catch of the cart in your analysis.

When I push the cart I do work on it, giving it KE. After the push, the cart continues up the incline. As it moves, it climbs, so it gains PEg. However, at the same time, it slows down so it loses KE. When it reaches its maximum displacement it comes to rest. At that point, all of its KE has now been converted to PEg. The cart now begins to move back toward its starting point, gaining speed and KE, and losing PEg. When it reaches the starting point, all of the energy is now back in the form of KE. I stopped the cart and in the process, the cart did work on my hand, losing all its KE while moving up and back. The total mechanical energy remained constant even though KE and PEg changed.
2. Perform a Linear Fit on the Mechanical Energy graph for Mechanical Energy only. Report the value of the slope. What is the significance of this slope?

The slope should have been very close to zero and negative. This would indicate that a small amount of ME is lost due to friction.

**Synthesis Questions**

Use available resources to help you answer the following questions.

A 100 kg roller coaster comes over the first hill at 2 m/sec \( (v_0) \). The height of the first hill \( (h) \) is 20 meters. See diagram below.

1. Use the conservation of energy to find the velocity of the roller coaster at point A. Use the methods we learned in class to show your work.

\[
\begin{align*}
M = 100 \text{ kg} \\
v_0 = 2 \text{ m/s} \\
h = 20 \text{ m} \\
v_A = ?
\end{align*}
\]

\[
\begin{align*}
(KE + PE)_i &= (KE + PE)_f \\
\frac{1}{2}mv_0^2 + mgh &= \frac{1}{2}mv_A^2 + mgh \\
v_A &= v_0 = 2 \text{ m/s}
\end{align*}
\]

2. Use the conservation of energy to find the velocity of the roller coaster at point B. Use the methods we learned in class to show your work.

\[
\begin{align*}
M = 100 \text{ kg} \\
v_0 = 2 \text{ m/s} \\
h = 20 \text{ m} \\
h_g = 10 \text{ m} = \frac{h}{2} \\
v_A = ?
\end{align*}
\]

\[
\begin{align*}
(KE + PE)_i &= (KE + PE)_f \\
\frac{1}{2}mv_0^2 + mgh &= \frac{1}{2}mv_B^2 + mgh \\
\frac{1}{2}mv_0^2 + 2mgh &= \frac{1}{2}mv_B^2 + mgh \\
v_B &= \sqrt{v_0^2 + gh} = \sqrt{(2 - 1)^2 + (9.8 \text{ m/s}^2)(20 \text{ m})} \\
v_B &= 14 \text{ m/s}
\end{align*}
\]
3. Use the conservation of energy to find the velocity of the roller coaster at point C. Use the methods we learned in class to show your work.

\[
\begin{align*}
V_0 &= \frac{2}{\eta} / s \\
h_i &= 20 m \\
V_c &= ? \\
\frac{1}{2} m V_c^2 + m g h_c &= \frac{1}{2} m V_0^2 + 0 \\
V_c &= \sqrt{V_0^2 + 2 g h_c} = \sqrt{(2 - 1/s)^2 + 2 (9, V_{i/s}) (20 m)} \\
V_c &= 20 m / s
\end{align*}
\]

4. Use the conservation of energy to find how high the roller will climb the last hill. Use the methods we learned in class to show your work.

\[
\begin{align*}
V_0 &= \frac{2}{\eta} / s \\
h_i &= 20 m \\
h_f &= ? \\
V_f &= 0 \\
h_f &= \frac{V_0^2}{2 g} + h_i \\
20 m &= \frac{(2 - 1/s)^2}{2 (9, V_{i/s})} + 20 m = [20.2 m] (or 20 m)
\end{align*}
\]

**Multiple Choice Questions**

Select the best answer or completion to each of the questions or incomplete statements below.

1. In our experiment, if we add 250 g to the mass of the cart, which of the following will be true?

   A. The maximum velocity of the cart will be greater.  
   B. The total energy in the system will be greater.  
   C. The kinetic energy will be less.  
   D. Gravity does less work to move the cart down the track.  
   E. None of these are true.
2. The car shown in the picture below (mass = 998 kg) has just run out of gas while moving at a velocity of 15 m/s. Assuming that friction is negligible, will the car make it to the gas station if it coasts the whole way? Choose the correct supporting answer.

\[ h_f = 10 \text{ m} \\
V_0 = 15 \text{ m/s} \\
h_o = 0 \\
V_f = ? \]

Elevation: 1,538 m, 1,542 m, 1,525 m, 1,535 m, 1,548 m

- **A.** No, the car does not have enough energy to get over the first hill.
- **B.** No, the car comes up just short of the gas station 2 m lower in elevation.
- **C.** Yes, with a velocity of 5.4 m/s.
- **D.** Yes, with a velocity of 3.8 m/s.
- **E.** Yes, with a velocity of 0.0 m/s

3. A square boulder with mass \( M \) slides down a frictionless ramp and hits another square boulder with mass \( 2M \) resting at the bottom of the ramp. If the two boulders stick together when they collide, how fast are they traveling after the collision?

\[ KE_i = KE_f + PE_f \]
\[ \frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + m g h_f \]
\[ v_f = \sqrt{v_0^2 - 2 g h_f} = \sqrt{(15 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(10 \text{ m})} \]

\[ v_f = 5.4 \text{ m/s} \]

- **A.** \( \sqrt{\frac{8gh}{3}} \)
- **B.** \( \sqrt{2gh} \)
- **C.** \( \sqrt{\frac{gh}{3}} \)
- **D.** \( \sqrt{\frac{2gh}{3}} \)
- **E.** Not enough information to answer this question.
Mechanical Energy v. Time for Cart Pushed up a Ramp

Linear Fit for: Latest | Mechanical Energy
ME = m(t+b)

m (Slope): 0.007245 +/- 0.000399
b (Y-Intercept): 0.4502 +/- 0.001038
Correlation: 0.9163
RMSE: 0.003155