Recall: The equation of a parabola that opens up or down: \((x-h)^2 = 4p(y-k)\)

For \(p\) positive, the graph opens up. For \(p\) negative, the graph opens down.

The equation of a parabola that opens right or left: \((y-k)^2 = 4p(x-h)\)

For \(p\) positive, the graph opens right. For \(p\) negative, the graph opens left.

Ex 1: The vertex of a parabola is \((-5,1)\) and the directrix is the line \(y = -2\). Find the focus of the parabola.

\[ V: (-5,1) \]
\[ D: y=-2 \]
\[ p = 3 \text{ units} \]
\[ F: (-5,1+3) \Rightarrow (-5,4) \]

*graph opens up since directrix below the vertex*

Ex 2: Find an equation of the parabola with a vertex at \((4,2)\) and a directrix at \(y = 5\).

\[ V: (4,2) \]
\[ D: y=5 \]
\[ p = 3 \text{ units} \]
\[ F: (4,2-3) \Rightarrow (4,-1) \]

\(\Rightarrow\) graph opens down \\
p is negative

Ex 3: Find an equation of the parabola with a focus at \((0, -2)\) and a directrix at \(x = 2\).

\[ F: (0, -2) \]
\[ D: x=2 \]
\[ p=1 \] (\(2p = 2\)) \(\Rightarrow\) graph opens left \\
p is negative

\[ V: (0+1, -2) \Rightarrow (1, -2) \]
Ex 4: Write the equation of the parabola with a vertex at the origin and a focus at (5,0).

V: (0,0)  \hspace{1cm} (y-k)^2 = 4p(x-h)
F: (5,0)  \hspace{1cm} (y-0)^2 = 4(5)(x-0)
p = 5 \text{ units}
D: \text{ opens right, } p \text{ is positive}
\hspace{1cm} y^2 = 20x

Ex 5: The 52-meter-long Hulme Arch Bridge in Manchester, England, is supported by cables suspended from a parabolic steel arch. The highest point of the arch is 25 meters above the bridge, and the focus of the arch is about 18 meters above the bridge.

Let the bridge be the x-axis and let the y-axis pass through the vertex of the arch.

a. Write an equation to model the arch.

\[(x-h)^2 = 4p(y-k)\]
\[(x-0)^2 = 4(-7)(y-25)\]
\[x^2 = -28(y-25)\]

b. Graph the equation.