ALG 1 – Unit 7 – Rational Exponents Day 1

Name

WARM UP: List the perfect squares under the following numbers.

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2	13^2	14^2	15^2
1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

BACKGROUND INFORMATION

A **Rational Number** is a real number that can be written as a simple fraction (i.e. as a **ratio**).

Example:

1.5 is a rational number because 1.5 = 3/2 (it can be written as a fraction)

$$1.5 = \frac{3}{2} \searrow \text{Ratio}$$

Rational

Number	As a Fraction	Rational?
5	5/1	Yes
1.75	7/4	Yes
.001	1/1000	Yes
0.111...	1/9	Yes
√2 (square root of 2)	?	**NO !**

Oops! The square root of 2 cannot be written as a simple fraction! And there are many more such numbers, and because they are not rational they are called <u>Irrational</u>.

Another famous irrational number is <u>Pi (π)</u>:

$$1.5 = \frac{3}{2} \searrow \text{Ratio}$$

Rational

$$\pi = 3.14159... = \frac{?}{?} \quad \text{(No Ratio)}$$

Irrational

> ### Formal Definition of Rational Number
> *A rational number is a number that can be in the form p/q where p and q are <u>integers</u> and q is not equal to zero.*

DISCOVERY

Using a calculator, determine the meaning of the exponent ½.

a) $25^{\frac{1}{2}} = 5$ b) $9^{\frac{1}{2}} = 3$ c) $100^{\frac{1}{2}} = 10$ d) $16^{\frac{1}{2}} = 4$ e) $49^{\frac{1}{2}} = 7$

> **CONCLUSION:**
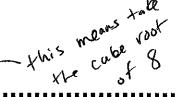> An exponent of ½ means   take the square root

Now try this problem: $(-36)^{\frac{1}{2}}$

What happened? Why do you think this happened? Explain your thoughts.

"ERROR"

or "non-real answer"

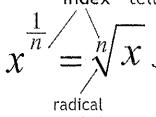There is no # you can square to get −36 because a + times a + is a + and a − times a − is a +

this means take the cube root of 8

> **PREDICTION:** Without using a calculator, figure out the answer to this problem.
>
> $8^{\frac{1}{3}} = 2$

rational exponents in the form $x^{\frac{1}{n}}$

index - tells what root you are taking

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

radical symbol

radicand - the number under the radical

Does the exponent of ½ really mean square root?

YES!!

$$\left(5^{\frac{1}{2}}\right)^2 = 5^1 = 5$$
$$\left(\sqrt{5}\right)^2 = 5$$

$$\left(5^{\frac{1}{2}}\right)^2 = \left(\sqrt{5}\right)^2 \Rightarrow 5^{\frac{1}{2}} = \sqrt{5}$$

Remember that squaring and square rooting are inverse operations. They undo one another. MathBits.com

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Examples:

Write the following in rational exponent form.

1. $\sqrt[4]{6} = 6^{\frac{1}{4}}$

2. $\sqrt[3]{x} = x^{\frac{1}{3}}$

3. $\sqrt{7} = 7^{\frac{1}{2}}$

4. $\sqrt[5]{3} = 3^{\frac{1}{5}}$

Write the following in radical form.

5. $2^{\frac{1}{9}} = \sqrt[9]{2}$

6. $5^{\frac{1}{2}} = \sqrt{5}$

7. $5^{\frac{1}{8}} = \sqrt[8]{5}$

8. $x^{\frac{1}{11}} = \sqrt[11]{x}$
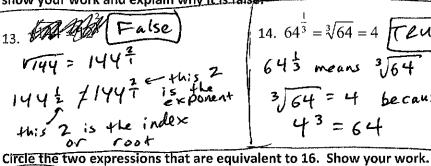
Simplify the following.

9. $121^{\frac{1}{2}} = 11$

10. $\left(9x^4\right)^{\frac{1}{2}} = 3x^2$

11. $27^{\frac{1}{3}} = 3$

12. $\left(81x^6\right)^{\frac{1}{2}} = 9x^3$

Determine whether each equation is _True_ or _False_. If it is true, show your work to defend your answer. If it is false, show your work and explain why it is false.

13. ~~√144 = 144~~ **False**

$\sqrt{144} = 144^{\frac{2}{2}}$

$144^{\frac{1}{2}} \neq 144^{\frac{2}{1}}$ ← this 2 is the exponent

this 2 is the index or root

14. $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$ **TRUE**

$64^{\frac{1}{3}}$ means $\sqrt[3]{64}$

$\sqrt[3]{64} = 4$ because

$4^3 = 64$

15. $\sqrt[4]{16} = 16 \cdot \frac{1}{4} = 4$ **FALSE**

$\sqrt[4]{16}$ means

$16^{\frac{1}{4}}$ not $16 \cdot \frac{1}{4}$.

$\sqrt[4]{16} = 2$ because $2^4 = 16$

Circle the two expressions that are equivalent to 16. Show your work.

$\left(64 \cdot 4\right)^{\frac{1}{2}}$ ⟵ circled

$(256)^{\frac{1}{2}}$

$= \boxed{16}$

$2 \cdot \sqrt{4^2}$

$= 2 \cdot 4$

$= 8$

$\sqrt{8^3}$

$= \sqrt{512}$

$\approx 22.627...$

$\left(8^{\frac{1}{3}}\right)^4$ ⟵ circled

~~8^{12}~~

$= (2)^4$

$= \boxed{16}$

VOCABULARY

The **square root** of a number is a value that, when multiplied by itself, gives the number.

Example: $4 \cdot 4 = 16$, so the square root of 16 is 4.

The symbol is $\sqrt{}$

Another example: $\sqrt{36} = 6$ (because $6 \cdot 6 = 36$)