Exponential Funtions
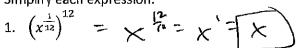
Students can:

✓ Target #1: Use the properties of exponents to write an equivalent form of an exponential function to reveal and explain specific information about rate of growth or decay

WARM-UP:

Simplify each expression.

1. $\left(x^{\frac{1}{12}}\right)^{12} = x^{\frac{12}{12}} = x^1 = \boxed{x}$

2. $\left(5^{\frac{1}{6}}\right)^{6x} = 5^{\frac{6x}{6}} = \boxed{5^x}$

MINI LESSON:

Exponential Growth:

$y = a \cdot (1 + r)^x$

$a = \underline{\text{Starting value}}$

$r = \underline{\% \text{ increase}}$

$x = \underline{\text{time}}$

Exponential Decay:

$y = a \cdot (1 - r)^x$

$a = \underline{\text{Starting value}}$

$r = \underline{\% \text{ decrease}}$

$x = \underline{\text{time}}$

Example 1
A town starts with 500 people and grows by 25% every year.

1. Choose the function that represents this situation.
   a. $y = 25 \cdot (500)^x$
   b. $y = 25 \cdot (0.500)^x$
   c. $y = 500 \cdot (0.25)^x$
   d. $y = 500 \cdot (1.25)^x$

2. At what rate does the number of people in the town increase by every month?

$$y = 500 \left(1.25^{\frac{1}{12}}\right)^{12x}$$

find this value $1.25^{\frac{1}{12}} = \boxed{1.02}$

or 102%

Example 2
A certain species of monkeys is dying off in a forest each year. The monkey population is currently 24,000 and is projected to decrease by 12% each year. $1 - .12 = .88$

1. Choose the function that represents this situation.
   a. $y = 24{,}000 \cdot (1.12)^x$
   b. $y = 88 \cdot (0.24000)^x$
   c. $y = 24{,}000 \cdot (0.88)^x$
   d. $y = 12 \cdot (24{,}000)^x$

2. What percent of the population is dying off each month?

$.88^{\frac{1}{12}} = .989$ or 99%

Example 3
You invest $1000 at a bank that pays an annual interest rate of 8%.

1. Choose the function that represents this situation.
   a. $y = 8 \cdot (1000)^x$
   b. $y = 1000 \cdot (1.08)^x$
   c. $y = 1000 \cdot (0.92)^x$
   d. $y = 8 \cdot (0.1000)^x$

2. What would your equivalent monthly interest rate be?

$1.08^{\frac{1}{12}} = 1.006$

Example 4

There are 2 mice in a barn. The number of mice is growing by 40% each year.

1. Choose the function that represents this situation.

   a. $y = 2 \cdot (1.4)^x$ *(circled)*
   b. $y = 2 \cdot (0.4)^x$

   c. $y = 40 \cdot (2)^x$
   d. $y = 40 \cdot (0.2)^x$

2. At what rate are the mice growing by every 6 months?

$$1.4^{\frac{6}{12}} = 1.18$$

WORKSHOP:

Example1

The population of the town Smithville in 2003 was estimated to be 35,000 people. The number of people in the town increases by 25% each year.

1. Choose the function that represents this situation.

   a. $y = 35,000 \cdot (0.25)^x$
   b. $y = 25 \cdot (35,000)^x$

   c. $y = 0.25 \cdot (35,000)^x$
   d. $y = 35,000 \cdot (1.25)^x$ *(circled)*

2. At what rate does the number of people in the town increase by each month?

$$1.25^{\frac{1}{12}} = 1.019$$

Example 2

A car is purchased for $14,000. The value of the car decreases by 33% each year.

1. Choose the function that represents this situation.

   a. $y = 14,000 \cdot (1.33)^x$
   b. $y = 0.67 \cdot (14,000)^x$

   c. $y = 14,000 \cdot (0.67)^x$ *(circled)*
   d. $y = 33 \cdot (14,000)^x$

2. Find the rate that the value of the car is depreciating by every 6 months.

$$.67^{\frac{6}{12}} = .819$$

Example 3

Marisa invests $300 at a bank that offers an annual interest rate of 15%.

1. Choose the function that represents this situation.

   a. $y = 15 \cdot (300)^x$
   b. $y = 300 \cdot (1.15)^x$ *(circled)*

   c. $y = 300 \cdot (0.85)^x$
   d. $y = 15 \cdot (0.300)^x$

2. What would Maria's equivalent monthly interest rate be?

$$1.15^{\frac{1}{12}} = 1.018$$

Exponential Funtions

1) Matt bought a new car at a cost of $25,000. Each year, the value of the car depreciates at a rate of 80%.
   1. Choose the function that represents this situation.
      a) $y = 80 \cdot (25,000)^x$
      b) $y = 25,000 \cdot (0.80)^x$
      c) $y = 25,000 \cdot (0.20)^x$
      d) $y = 0.20 \cdot (25,000)^x$
   2. At what rate is the car depreciating each month?

$$.20^{\frac{1}{12}} = .87$$

2) Jose invests $600 at a bank offering an annual interest rate of 10%.
   1. Choose the function that represents this situation.
      a) $y = 600 \cdot (1.10)^x$
      b) $y = 10 \cdot (600)^x$
      c) $y = 600 \cdot (0.10)^x$
      d) $y = 600 \cdot (10)^x$
   2. What would the equivalent monthly interest rate be?

$$1.10^{\frac{1}{12}} = 1.008$$

3) You invest $2400 at a bank that pays an annual interest rate of 7%.
   1. Choose the function that represents this situation.
      a) $y = 7 \cdot (2400)^x$
      b) $y = 2400 \cdot (1.07)^x$
      c) $y = 2400 \cdot (0.07)^x$
      d) $y = 0.07 \cdot (2400)^x$
   2. What would the equivalent interest rate be for every 6 months?

$$1.07^{\frac{6}{12}} = 1.0344$$

4) In 1985, there were 300 cell phone subscribers in the town of Centerville. The number of subscribers increased by 18% each year.
   1. Choose the function that represents this situation.
      a) $y = 300 \cdot (1.18)^x$
      b) $y = 300 \cdot (0.18)^x$
      c) $y = 300 \cdot (0.82)^x$
      d) $y = 18 \cdot (300)^x$
   2. At what rate did the number of subscribers increase for every 6 months?

$$1.18^{\frac{6}{12}} = 1.086$$

5) You have inherited land that was purchased for $20,000 in 1960. The value of the land increased by a rate of 6% each year.
   1. Choose the function that represents this situation.
      a) $y = 20,000 \cdot (0.06)^x$
      b) $y = 6 \cdot (20,000)^x$
      c) $y = 20,000 \cdot (6)^x$
      d) $y = 20,000 \cdot (1.06)^x$
   2. How much would the value of the land be increasing by every 3 months?

$$1.06^{\frac{3}{12}} = 1.015$$

6) Howard decided to collect Star Wars figurines. He started off with 2 figurines and collects 5 more each week. Write a function rule to describe the situation. Is this an example of a linear or exponential function?

Weeks	# of figurines
0	2
1	7
2	12
3	17
4	22

+5
+5
+5
+5

$$y = 5x + 2$$

rate of change

starting value

7) Peggy started off the year with $15,000. She spends half of the money remaining in her account each month and does not put any money back. Write a function rule to describe the situation. Is this an example of a linear or exponential function? How long will it take Peggy to spend all of her money?

Months	$ in account
0	15000
1	7500
2	3750
3	1875
4	937.5

$\times \frac{1}{2}$
$\times \frac{1}{2}$
$\times \frac{1}{2}$

$$y = 15000 \cdot \frac{1}{2}^{x}$$

starting value

decay factor

she will never be able to spend all of her money. there is no "x-intercept"

8) A scientist was studying the rate of bacteria increase in a culture they had taken. The culture contained 200 bacteria when the study began. The scientist noticed the bacteria growing at a rate of 4 bacteria per second. Write a function rule to describe the situation. Is this an example of a linear or exponential function?

Seconds	bacteria
0	200
1	204
2	208
3	212
4	216

+4
+4
+4
+4

$$y = 4x + 200$$

rate of change

starting value

9) On Halloween night, Sam was passing out candy. She started out with 225 pieces of candy. Being the generous person she was, she gave each trick-or-treater 5 pieces. Write a function rule to describe the situation. Is this an example of a linear or exponential function? How many trick-or-treaters will be able to stop at Sam's house?

Trick-or-treaters	candy
0	225
1	220
2	215
3	210
4	205

−5
−5
−5
−5

$$y = -5x + 225$$

rate of change

starting value

$$-5x + 225 = 0$$
$$-5x = -225$$
$$x = 45$$

45 trick-or-treaters can stop by