

Worksheet 09.04c

Find the partial fraction decomposition of each.

$$1) \quad \frac{5x-4}{x^2-x-2} = \frac{5x-4}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$= \frac{2}{x-2} + \frac{3}{x+1}$$

$$5x-4 = A(x+1) + B(x-2)$$

$$x=2$$

$$x=-1$$

$$6 = 3A$$

$$-9 = -3B$$

$$2 = A$$

$$3 = B$$

$$2) \quad \frac{2x^2+x-50}{x^2-x-12} \Rightarrow \frac{3x-26}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$

$$= 2 + \frac{-2}{x-4} + \frac{5}{x+3}$$

$$3x-26 = A(x+3) + B(x-4)$$

$$x=4$$

$$x=-3$$

$$-14 = A(7)$$

$$-35 = -7B$$

$$-2 = A$$

$$5 = B$$

$$x^2 - x - 12 \begin{array}{r} 2 \\ \hline 2x^2 + x - 50 \\ 2x^2 - 2x - 24 \\ \hline 3x - 26 \end{array}$$

$$3) \quad \frac{5x-11}{2x^2+x-6} = \frac{5x-11}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$$

$$5x-11 = A(x+2) + B(2x-3)$$

$$x=2$$

$$x = \frac{3}{2}$$

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$$-21 = A(0) + B(-7)$$

$$\frac{-7}{2} = A \frac{7}{2} + B(0)$$

$$3 = B$$

$$-1 = A$$

$$= \frac{-1}{2x-3} + \frac{3}{x+2}$$

$$4) \frac{5x-1}{x^3-x} = \frac{5x-1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{D}{x-1}$$

$$5x-1 = A(x+1)(x-1) + Bx(x-1) + Dx(x+1)$$

$$x=0$$

$$-1 = A(1)(-1)$$

$$1 = A$$

$$x=1$$

$$4 = D(1)(2)$$

$$2 = D$$

$$x=-1$$

$$-6 = B(-1)(-2)$$

$$-6 = 2B$$

$$-3 = B$$

$$\frac{1}{x} + \frac{-3}{x+1} + \frac{2}{x-1}$$

$$5) \frac{3x^2+21x-84}{x^3-4x^2-11x+30} = \frac{3x^2+21x-84}{(x-2)(x+3)(x-5)} = \frac{A}{x-2} + \frac{B}{x+3} + \frac{D}{x-5}$$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & & 2 & -4 & -30 \\ \hline -3 & 1 & -2 & -15 & 0 \\ & & -3 & 15 & \\ \hline 5 & 1 & -5 & 0 & \\ & & 5 & & \\ \hline & 1 & 0 & & \end{array}$$

$$3x^2+21x-84 = A(x+3)(x-5) + B(x-2)(x-5) + D(x-2)(x+3)$$

$$x=2$$

$$-30 = A(5)(-3)$$

$$-30 = -15A$$

$$2 = A$$

$$x=-3$$

$$-120 = B(-5)(-8)$$

$$-3 = B$$

$$x=5$$

$$96 = D(3)(8)$$

$$96 = 24D$$

$$4 = D$$

$$= \frac{2}{x-2} + \frac{-3}{x+3} + \frac{4}{x-5}$$