

Worksheet 09.04b

Find the partial fraction decomposition of each.

$$1) \quad \frac{-5x+4}{x^2-x} = \frac{-5x+4}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} = \frac{-4}{x} + \frac{-1}{x-1}$$

$$-5x+4 = A(x-1) + Bx$$

$$x=0 \qquad \qquad \qquad x=1$$

$$4 = -A \qquad \qquad -1 = B$$

$$-4 = A$$

$$2) \quad \frac{3x+10}{x^2+9x+20} = \frac{3x+10}{(x+5)(x+4)} = \frac{A}{x+5} + \frac{B}{x+4} = \frac{5}{x+5} + \frac{-2}{x+4}$$

$$3x+10 = A(x+4) + B(x+5)$$

$$x=-4 \qquad \qquad \qquad x=-5$$

$$-2 = B(1) \qquad \qquad -5 = A(-1)$$

$$3) \quad \frac{2x^2-9x-10}{x^2-5x} = 2 + \frac{x-10}{x(x-5)} = \frac{A}{x} + \frac{B}{x-5} = 2 + \frac{2}{x} + \frac{-1}{x-5}$$

$$x-10 = A(x-5) + Bx$$

$$x=0 \qquad \qquad \qquad x=5$$

$$-10 = A(-5) \qquad \qquad -5 = 5B$$

$$2 = A \qquad \qquad -1 = B$$

$$\begin{array}{r} 2 \\ x^2 - 5x + 0 \overline{) 2x^2 - 9x - 10} \\ \underline{2x^2 - 10x + 0} \\ x - 10 \end{array}$$

$$4) \frac{-6x^2 + 3x + 5}{x^3 - x} = \frac{-6x^2 + 3x + 5}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$= -6x^2 + 3x + 5 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$$

$x=0$	$x=-1$	$x=1$
↓	↓	
$5 = A(1)(-1)$	$-4 = B(-1)(-2)$	$2 = C(1)(2)$
$-5 = A$	$-2 = B$	$1 = C$

$$\frac{-6x^2 + 3x + 5}{x^3 - x} = \frac{-5}{x} + \frac{-2}{x+1} + \frac{1}{x-1}$$

$$5) \frac{-2x^2 + 19x - 13}{(x-5)(x-1)(x-1)^2} = \frac{A}{x-5} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^2} =$$

$$-2x^2 + 19x - 13 = A(x-1)^2 + B(x-5)(x-1) + C(x-5)$$

LET $x=5$	LET $x=1$
↓	↓
$32 = A(4)^2$	$4 = C(-4)$
$32 = 16A$	$-1 = C$
$2 = A$	

$$-2x^2 + 19x - 13 = 2(x-1)^2 + B(x-5)(x-1) - 1(x-5)$$

$$\text{LET } x=0$$

↓

$$-13 = 2(-1)^2 + B(-5)(-1) - 1(-5)$$

$$-13 = 2 + 5B + 5$$

$$-20 = 5B$$

$$-4 = B$$

$$\frac{-2x^2 + 19x - 13}{(x-5)(x-1)(x-1)^2} = \frac{2}{x-5} + \frac{-4}{(x-1)^2} + \frac{-1}{(x-1)^2}$$