Precalculus 4.1 Polynomial Functions (x_1, y_1) (x_2, y_2)

Objective: able to identify polynomial functions & their degree; graph using transformations; identify the real zeroes & their multiplicity; analyze the graph; and build cubic models from data

A **polynomial function** is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0 x^0$

where $a_n, a_{n-1}, \cdots, a_1, a_0$ are real numbers and n is a nonnegative integer.

The domain is the set of all real numbers.

$x^{-1} = \frac{1}{x}$

$x^{\frac{1}{2}} = \sqrt{x}$

What is a second degree polynomial function? * $(a_2x^2 + a_1x^1 + a_0)$

$y = a_2x^2 + a_1x^1 + a_0$

What is a first degree polynomial function? * * $(a_1x^1 + a_0)$

$y = a_1x^1 + a_0$

What is a zero degree polynomial function? * (a_0)

$y = b = a_0$

What is a no degree polynomial function? * 0

$y = 0$

Oct 5-11:52 AM

State which are polynomial functions and state the degree for those that are.

*Graphs of polynomial functions should be smooth and continuous.

1. $f(x) = 6 - 2x^6$ degree = 6

~~2.~~ $g(x) = \sqrt[4]{x}$

~~3.~~ $h(x) = x^2 - 1 \;/\; x^3 + 2$ $\frac{x^2-1}{x^3+2}$

4. $k(x) = 0$ no degree

5. $H(x) = 10$ degree = 0

6. $G(x) = 4x(x-1)^2$ degree = 3

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Power function = ax^n : Graph these functions on the same screen

$f(x) = x^2$

$g(x) = x^4$

$h(x) = x^8$

$k(x) = x^{12}$

$m(x) = x^{20}$

Comments?

even exponents have same end behavior

as the exponent grows the graph gets more vertical & flattens at origin

same for all

-1	1
0	0
1	1

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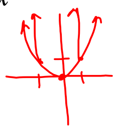
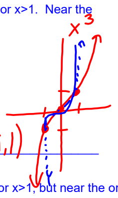
A **power function of degree n** is a monomial of the form $f(x) = ax^n$

where a is a real number, $a \neq 0$, and $n > 0$ is an integer.



Characteristics of a power function with **even** exponent, n.

1. graph is symmetric about the y-axis, so f is even
2. Domain is the set of all real numbers. The range is the set of nonnegative real numbers.
3. The graph always contains the points $(-1,1)$, $(0,0)$, $(1,1)$
4. As n increases the graph becomes more vertical when x<-1 or x>1. Near the origin the graph tends to flatten



Characteristics of a power function with **odd** exponent, n.

1. Graph is symmetric about the origin, so f is odd
2. Domain and range are $\mathbb{R}$
3. The graph always contains the points $(-1,-1)$ $(0,0)$ $(1,1)$
4. As n increases the graph becomes more vertical when x<-1 or x>1, but near the origin the graph tends to flatten.

Oct 5-12:39 PM

Graph each of the following functions. Explain the transformations with respect to their parent functions.

1. $f(x) = (x-2)^5$

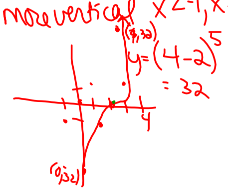
parent $y = x^3$

Right 2

flatten at ~~vertex~~ ? (2,0)

more vertical $x < -1, x > 1$

$y = (4-2)^5$
$= 32$



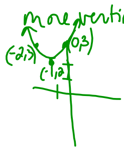
2. $f(x) = (x+1)^4 + 2$

left 1

up 2

flatten at ? (vertex)

more vertical $x < -1, x > 1$



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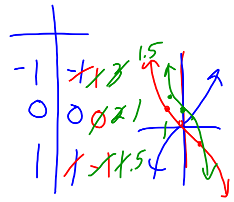
3. $f(x) = 2 - 0.5x^3$

$= -.5x^3 + 2$

Vertical reflection over x axis

↑ 2

Vertical compression by a factor of .5



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If f is a polynomial function and r is a real number for which $f(r)=0$ then r is called a **real zero of f**.

$x^2=0$

4. Form a third degree polynomial function with zeros at -2, 2, and 3. Use a graphing utility to verify.

(4,7)

$f(x)=a(x+2)^1(x-2)^1(x-3)^1$

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These statements are equivalent

1. r is a real zero of a polynomial function f.
2. r is an x-int. of the graph of f.
3. x-r is a factor of f.
4. r is a solution to the equation f(x) = 0.

So the real zeros of a graph are the x-intercepts, and found by solving the equation f(x) = 0.

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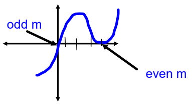
If the same factor $(x-r)$ occurs more than once, then r is called a repeated (or multiple) zero of f. So, if $(x-r)^m$ is a factor of a polynomial function f and $(x-r)^{m+1}$ is not a factor of f, then r is called a **zero of multiplicity m of f.**



5. Form a fourth degree polynomial function with zeros at -1, 2, and 3 with multiplicity 2. Use a graphing utility to verify.

Oct 28-2:00 PM

What does the role of multiplicity of zeros have to do with the graph of a polynomial function?

Oct 28-2:02 PM

6. List each zero and its multiplicity for the following polynomial function:

$f(x)=2(x+3)^2(x+1)^3$. Determine the behavior of the graph at each zero.

Use a graphing utility to verify.

Oct 28-2:03 PM

Local maxima or local minima are points on a graph where the graph changes from an increasing to a decreasing function, or vice versa. These points are called **turning points.**

How many turning points can a graph have?

* if the degree is n, the graph has at most n-1 turning points.

The behavior of the graph of a polynomial function for large absolute values of x is referred to as its **end behavior.** So, for large absolute values of x, the graph of the polynomial function

$f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$ resembles the graph of the power function

$f(x)=a_nx^n$

What are the four cases?

$n\geq 2$(even), $a_n>0$	$n\geq 2$(even), $a_n<0$	$n\geq 3$(odd), $a_n>0$	$n\geq 2$(odd), $a_n>0$

Oct 28-2:11 PM

For the following polynomial functions:
a. Determine the end behavior
b. Find the x - and y -intercepts
c. Determine whether each x -intercept is of odd or even multiplicity
d. Find the power function that the graph of f resembles for large values of $|x|$
e. Determine the maximum number of turning points
f. Graph using a graphing utility
g. If they exist, determine the local maxima and local minima to the nearest hundredths.
h. Find the domain and range
i. Use the graph to determine where the function is increasing and decreasing

7. $f(x)=x(x+2)^2$

Oct 28-3:09 PM

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g. If they exist, determine the local maxima and local minima to the nearest hundredths.
h. Find the domain and range
i. Use the graph to determine where the function is increasing and decreasing

8. $f(x)=x^3-0.8x^2-4.67x+3.73$

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9. The table below lists the decade, x, and the number of major hurricanes, y, that hit the U.S.

Decade, x	Major Hurricanes, y
1921-1930, 1	5
1931-1940, 2	8
1941-1950, 3	10
1951-1960, 4	8
1961-1970, 5	6
1971-1980, 6	4
1981-1990, 7	5
1991-2000, 8	5

a. Use a graphing utility to create a scatter plot.

b. Use a graphing utility to create a cubic function of best fit (CUBIC REGression)

c. Use the cubic function from part (b) to predict the number of hurricanes to hit the U.S. from 2001-2010.

Nov 3-7:20 PM

Rate yourself on how well you understood this lesson.

I don't get it at all	I sort of get it	I understand most, but I need more practice	I understand it pretty well	I got it!!
1	2	3	4	5

What I still need to work on....

Aug 24-10:50 AM