Precalculus 5.5 Properties of Logarithms

Objective: able to work with the properties of logs; expand a log expression as a sum/difference; condense a log expression to a single log; evaluate/graph logs for any base

Logarithms have some very useful properties that can be derived directly from the definition and the laws of exponents.

$\log_a x$ $\quad \log_a(M+N) \neq \log_a(MN)$

Complete the following properties of logarithms:

1. $\log_a 1 = 0$ $\quad a^0 = 1$
2. $\log_a a = 1$ $\quad a^1 = a$
3. $a^{\log_a M} = M$
4. $\log_a a^r = r$
5. $\log_a (MN) = \log_a M + \log_a N$
6. $\log_a (M+N) = \log_a(M+N)$
7. $\log_a (M-N) = \log_a(M-N)$
8. $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$
9. $\log_a\left(\frac{1}{N}\right) = -\log_a N = \log_a 1 - \log_a N = 0 - \log_a N$
10. $\log_a (M^r) = r(\log_a M)$

If M, N, and a are positive real numbers, with $a \neq 1$, then the following is true: $M = N$ if and only if $\log_a M = \log_a N$.

This means that we can 'take the log of both sides' of an equation, which we will do in the next section.

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#8

let: $M = a^x \to \log_a M = x$

$N = a^y \to \log_a N = y$

$$\log_a\left(\frac{M}{N}\right) = \log_a \frac{a^x}{a^y} = \log_a(a^{x-y}) = x - y = \log_a M - \log_a N$$

$$\boxed{a^{\log_a M} = M}$$

$f \circ f^{-1} = x$

$f(f^{-1}(x)) = a^{\log_a x} = x$

$f(x) = a^x$

$y = a^x$

$x = a^y$ as log $\quad \log_a x = y$ $f^{-1}(x)$, $x > 0$

#5

$x^2 \cdot x^3 = x^{2+3}$

$\log_a(MN)$

let: $M = a^x$

$N = a^y$

substitute $\log_a(a^x \cdot a^y) = \log_a(a^{x+y}) = x+y$

$\log_a M = x$

$\log_a N = y$ } substitute $\to = \log_a M + \log_a N$

Prove #3 of the preceding properties.

If $M = N$ then $\log_a(M) = \log_a(N)$

Logarithms can be used to transform products into sums, quotients into differences, and powers to factors.

$\log x = \log_{10} x$

1. Write $\log\left[\frac{x^3\sqrt{x+1}}{(x-2)^2}\right]$ as a sum and / or difference of logarithms, and express powers as factors. (10)

$= \log(x^3\sqrt{x+1}) - \log(x-2)^2$

$= \log x^3 + \log(x+1)^{1/2} - \log(x-2)^2$

$= 3(\log x) + \frac{1}{2}(\log(x+1)) - 2(\log(x-2))$

$= 3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)$

2. Write $21\log_3\sqrt[3]{x} + \log_3(9x^2) - \log_3 25$ as a single logarithm.

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2. Write $2\log_3 \sqrt[3]{x} + \log_3(9x^2) - \log_3 25$ as a single logarithm.

$\log_3(ax)^2$

$\log_3 (x^{\frac{1}{3}})^2$

$\log_3(x^{\frac{2}{3}} \cdot 9x^2) - \log_3 25$

$\boxed{\log_3\left(\frac{9x^{9}}{25}\right)}$

To calculate logarithms having a base other than 10 or e, you use the **change of base formula**. If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then

$\log_a M = \frac{\log_b M}{\log_b a}$ and $\log_a M = \frac{\log M}{\log a}, b = 10$ and $\log_a M = \frac{\log_e M}{\log_e a}, b = e$

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To calculate logarithms having a base other than 10 or e, you use the **change of base formula**. If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then

$\log x = \log_{10} x$

$\ln x = \log_e x$

$\log_a M = \frac{\log_b M}{\log_b a}$ and $\log_a M = \frac{\log M}{\log a}, b = 10$ and $\log_a M = \frac{\log_e M}{\log_e a}, b = e$

$\log_{\square} \square$

Derive the change in base formula:

$\log_{12} 10 = \frac{\log 10}{\log 12}$

$y = \log 12$

$10^y = 12$

Calculate to the thousandths place.

3. $\log_5 18$

$= \frac{\log 18}{\log 5} \approx 1.796$

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4. $\log_\pi \sqrt{2}$

$= \frac{\log \sqrt{2}}{\log \pi} \approx .303$

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Change of base

$y = \log_a M$

$a^y = M$ (write as exp)

$\log_b a^y = \log_b M$ (take log of both sides, move exp. to factor)

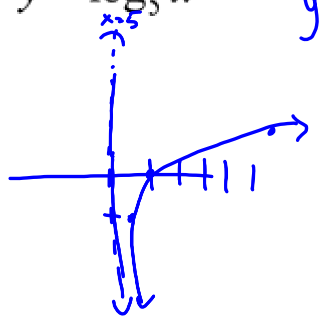
$y(\log_b a) = \log_b M$

$y = \frac{\log_b M}{\log_b a}$

5. Graph

$y = \log_5 x$

$y = \frac{\log x}{\log 5}$



$\frac{1}{5}$	-1
1	0
5	1

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Rate yourself on how well you understood this lesson.

I don't get it at all	I sort of get it	I understand most, but I need more practice	I understand it pretty well	I got it!!
1	2	3	4	5

What I still need to work on....

If $M = N$ then $\log M = \log N$

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