Precalculus 5.4 Logarithmic Functions

Objective: able to switch b/w logarithmic & exponential statements; evaluate logarithmic expressions; determine domain & range; graph & solve logarithmic equations

Find the inverse function of $f(x) = a^x$.

$y = a^x$

Switch x + y

$x = a^y$ exponent

value base

or

exponent value

$\boxed{y = \log_a x}$

base

inverse of exp.

The **logarithmic function to the base a**, where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a x$ (read as "y is the logarithm to the base a of x") and is defined by

$y = \log_a x$ if and only if $x = a^y$.

The domain of the logarithmic function $y = \log_a x$ is $x > 0$.

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1. Change the exponential expression $16 = 4^2$ to an equivalent logarithmic expression.

$y = \log_a x$

$2 = \log_4 16$

2. Change the logarithmic expression $\log_3 \frac{1}{9} = -2$ to an equivalent exponential expression.

$3^{-2} = \frac{1}{9}$

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2. Change the logarithmic expression $\log_3 \frac{1}{9} = -2$ to an equivalent exponential expression.

3. Find the exact value of $\log_5 125$.

$y = \log_5 125$

$5^3 = 125$

$\boxed{y = 3}$

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3. Find the exact value of $\log_5 125$.

4. Find the domain of $H(x) = \log_5 x^3$.

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4. Find the domain of $H(x) = \log_5 x^3$.

$x^3 > 0$

$x > \sqrt[3]{0}$

$\{x \mid x \in \mathbb{R},\ x > 0\}$

5. Find the domain of $G(x) = \log_3 \left(\frac{x}{x-1}\right)$

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5. Find the domain of $G(x) = \log_3 \left(\frac{x}{x-1}\right)$

$\frac{x}{x-1} > 0$

$x \neq 1$

$\frac{-1}{-1-1} > 0$ ✓

$\frac{.5}{.5-1} \not> 0$

$x \neq 0$ $\frac{2}{2-1} > 0$ ✓

-1 0 .5 1 2

$\{x \mid x \in \mathbb{R},\ x < 0,\ x > 1\}$

6. Graph $f(x) = \log_a x$, $a > 0$ and $a \neq 1$. What do you observe?

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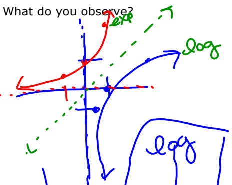
$y = \log_a x$

$x = a^y$

inverse of log $y = a^x$

$\log x = \log_{10} x$



	$x=0$
-1	$\frac{1}{a}$
0	1
1	a

log

1/a	-1
1	0
a	1

one to one

Smooth + continuous

increasing $(x > 0)$

D: $x > 0$

R: $\mathbb{R}$

Vert. asym $x = 0$

The logarithmic function to the **base e is the natural logarithmic function**, where $\log_e x = \ln x$ and is defined by

$y = \ln x$ **if and only if** $x = e^y$.

The domain of the natural logarithmic function $y = \ln x$ is $x > 0$.

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7. What does the graph of $f(x) = \ln x$ look like?

$y = \ln x$

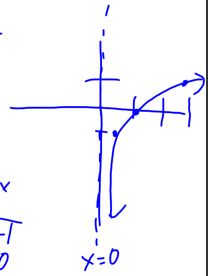
$e^y = x$

inverse $y = e^x$

-1	1/e
0	1
1	$e \approx 2.718$

ln x

$\frac{1}{2.718} = 1/e$	-1
1	0
e	1



8. Graph $g(x) = \ln(x - 3)$ by using transformations of the graph of $f(x) = \ln x$. Determine the domain, range, and vertical asymptote.

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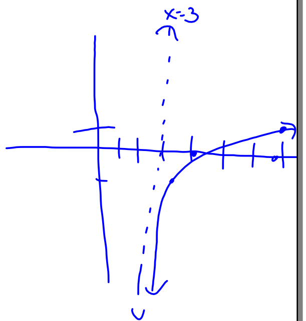
$\log_e x = \ln x$

ln x	
1/e	-1
1	0
e	1

+3	
$3+\frac{1}{e}$	-1
4	0
3+e	1

$-(\ln(x-3))$

reflect over x axis



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Graph

$y = \log_4(x-4) + 1$

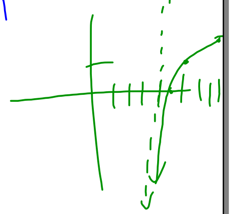
$y - 1 = \log_4(x-4)$

$4^{y-1} = x - 4$

	$\log_4 x$
$\frac{1}{4}$	-1
1	0
4	1

+4	
$4\frac{1}{4}$	-1
5	0
8	1

	+1
$4\frac{1}{4}$	0
5	1
8	2



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Nov 13-2:43 PM

Equations that contain logarithms are called logarithmic equations.

9. Solve $\log_5 x = 3$

$5^3 = x$

$\boxed{125 = x}$

10. Solve $\log_x\left(\frac{1}{8}\right) = 3$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{1}{8}}$

$x = \frac{1}{2}$

$x = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}$

or

$x = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} \cdot \frac{\sqrt[3]{8} \cdot \sqrt[3]{8}}{\sqrt[3]{8} \cdot \sqrt[3]{8}}$

$= \frac{\sqrt[3]{64}}{8}$

$= \frac{4}{8}$

$\sqrt[3]{64}$, $16 \cdot 4$, $4 \cdot 4 \cdot 4$

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11. Solve $e^{-2x} = \frac{1}{3}$

$\ln_e e = 1$

$\log_e$

$$\frac{\ln \frac{1}{3}}{-2} = \frac{-\cancel{2}x}{-\cancel{2}}$$

calc

$$x = \frac{\ln \frac{1}{3}}{-2} \approx .549$$

12. A model for the number of people N in a college community who have heard a certain rumor is

$$N = P\left(1 - e^{-0.15d}\right)$$

where P is the total population of the community and d is the number of days that have elapsed since the rumor began. In a community of 1000 students, how many days will elapse before 450 students have heard the rumor?

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12. A model for the number of people N in a college community who have heard a certain rumor is

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P d N

$$\frac{45\cancel{0}}{100\cancel{0}} = \frac{100\cancel{0}\left(1 - e^{-.15d}\right)}{100\cancel{0}}$$

$$\frac{45}{100} - 1 = 1 - 1 - e^{-.15d}$$

$$\frac{\frac{45}{100} - 1}{-1} = \frac{-e^{-.15d}}{-1}$$

$$-\frac{45}{100} + 1 = e^{-.15d}$$

$$\ln\left(-\frac{45}{100} + \frac{100}{100}\right) = -.15d$$

$$\frac{\ln\left(\frac{55}{100}\right)}{-.15} = \frac{\cancel{-.15}d}{\cancel{-.15}}$$

4 days ≈ d

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Rate yourself on how well you understood this lesson.

I don't get it at all	I sort of get it	I understand most, but I need more practice	I understand it pretty well	I got it!!
1	2	3	4	5

What I still need to work on....

Aug 24-10:50 AM