(95) $f(x) = 2x^4 + 12x^3 - 8x^2 - 48x$

① ↖ ↗

② y int (0,0) $f(x) = 2x(x^3 + 6x^2 - 4x - 24)$

x-int $= 2x(x^2(x+6) - 4(x+6))$

(0,0) (-6,0) (-2,0) (2,0) $= 2x((x+6)(x^2-4))$

③ zeros 0, -6, -2, 2 $2x(x+6)(x+2)(x-2)$

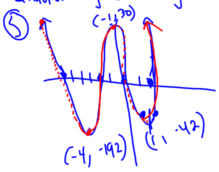
multiplicity 1, 1, 1, 1

behavior goes through each

⑤ $y = -8(2)(-2)(-6) = -192$

$= -2(5)(1)(-3) = 30$

$= 2(7)(3)(-1) = -42$



⑦ D: ℝ

R $(-192, \infty)$

Dec $(-\infty, -4)$ $(-1, 1)$

Inc $(-4, -1)$ $(1, \infty)$

Oct 25-2:44 PM

Precalculus 4.2 Rational Functions

Objective: able to find the domain; find the vertical asymptotes; find the horizontal or oblique asymptotes of a rational function

A **rational function** is a function of the form $R(x) = \frac{p(x)}{q(x)}$

, where p and q are polynomial functions and q is

not the zero polynomial. The domain is the set of all real numbers except those which make $q(x) = 0$.

$R(x) = \frac{p(x)}{q(x)}$

Find the domain of the following rational function.

1. $k(x) = \frac{x}{x^4 - 81}$

$x^4 - 81 = 0$

$(x^2+9)(x^2-9) = 0$

$(x^2+9)(x+3)(x-3) = 0$

$x^2+9=0$ $x+3=0$ $x-3=0$

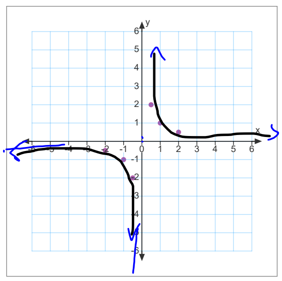
$x^2 = -9$ $x \neq -3$ $x \neq 3$

not real

$\{x \mid x \in \mathbb{R}, x \neq \pm 3\}$

Nov 3-7:27 PM

2. What does the graph of look like? Analyze the graph.



* as x $\to -\infty$, f(x) $\to 0$ $f(x) = \frac{1}{x}$

lim f(x) = 0

$x \to \infty$, $y \to 0$

$x \to -\infty$, $y \to 0$

* as x $\to \infty$, f(x) $\to 0$

lim f(x) = 0

$x \to 0^-$, $y \to -\infty$

$x \to 0^+$, $y \to \infty$

Domain $\{x \mid x \in \mathbb{R}, x \neq 0\}$

~~as x → 0⁻, f(x)~~

~~as x → 0⁺, f(x)~~

~~Domain =~~

$\lim_{x \to \infty} \frac{1}{x} = 0$

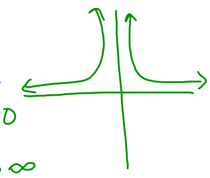
asymptotes? $x = 0$ (vertical) $y = 0$ (horizontal)

f(0) does not exist in the reals and f(x)≠0.

symmetric about origin $-f(x) = f(-x)$

Nov 3-8:05 PM

3. What does the graph of $g(x) = \frac{1}{x^2}$ look like? Analyze the graph. $\to g(x) = \text{positive}$



as x approaches ∞, f(x) approaches $y \to 0$

as x approaches −∞, f(x) approaches $y \to 0$

as x approaches ⁻0, f(x) approaches $y \to \infty$

as x approaches ⁺0, f(x) approaches $y \to \infty$

Domain $\{x \mid x \in \mathbb{R}, x \neq 0\}$

Nov 3-8:12 PM

4. Use transformations to graph

$\frac{1}{x^2}$ parent $h(x) = \frac{1}{(x+2)^2} - 3$

transformations

y-int. $y = \frac{1}{(0+2)^2} - 3 = \frac{1}{4} - 3 = -2\frac{3}{4}$ $(0, -2\frac{3}{4})$ $(x^2 - 4)$

1. ← 2

2. ↓ 3

x-int $0 = \frac{1}{(x+2)^2} - 3$

Domain

$3 = \frac{1}{(x+2)^2}$

$(x+2)^2 = 0$

$3(x+2)^2 = 1$

$x + 2 = 0$

$-2 \quad -2$

$x = -2$

$(x+2)^2 = \frac{1}{3}$

$x + 2 = \pm\sqrt{\frac{1}{3}}$

$\{x \mid x \in \mathbb{R}, x \neq -2\}$

$x = -2 \pm \sqrt{\frac{1}{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$-2 \pm \frac{\sqrt{3}}{3}$

$(-2 \pm \frac{\sqrt{3}}{3}, 0)$

Asymptotes

lim $1/(x+2)^2 - 3$ = ______
x → ∞

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Asymptotes

If, as $x \to \infty$ or as $x \to -\infty$, the values of the rational function R approach some fixed number L, then the line $y = L$ is a **horizontal asymptote** of the graph of R.

If, as x approaches some real number c [as $x \to c$, $c \in \Re$], the values of $|R(x)| \to \infty$ then the line $x = c$ is a **vertical asymptote** of the graph of R.

If an asymptote is neither horizontal or vertical, then it is called an **oblique asymptote**.

How do you **find vertical asymptotes**? (behavior of the graph when x is close to some number,c)

$R(x) = \frac{p(x)}{q(x)}$,in lowest terms, has a vert. asym. x = r if r is a real zero of the denominator q. (x - r is a factor of the denominator q)

the graph of R Never intersects a vertical asym.

Nov 3-8:23 PM

How do you **find horizontal asymptotes**? (describes the end behavior of the graph as x → ∞ or as x → -∞)

* Proper function= the degree of the numerator (n) is less than the degree of the denominator (m), and as x approaches ∞ or −∞ the function approaches 0. Thus; the line $y=0$ is a horizontal asymptote.

$n<m$

* the graph of a function may cross a horiz. asym.

2 times for horizontal asym.

n is the degree of p(x), m is the degree of q(x)

1) if n < m,

$$\frac{x-1}{x^2-4} \quad n=1 \quad m=2$$

since $n<m$, horizontal asym $y=0$

2) if n = m, use long division (short cut = lead coef. of p(x) / lead coef. q (x)) to find the constant,L)

$$\frac{4x^2-1}{2x^2-4} \quad n=2 \quad m=2$$

since $n=m$, horizontal asym $y=L$

$y=\frac{4}{2}$

$y=2$

Nov 3-8:27 PM

How do you **find oblique asymptotes**?

* when as x approaches ∞ or −∞, the value of the function approaches a linear expression, $y=mx+b$ $m\neq 0$

* the graph of a function may intersect an oblique asymptote

$n>m$

1) If $n = m+1$, use long division to find the expression ax+b--

* $y=ax+b$ is an oblique asymptote.

2) If n ≥ m+2, use long division, the quotient is of degree 2 or higher, so * no horizontal or oblique asym.

$$\frac{4x^8}{x^5-4} \quad n>m$$

$4x^3 \rightarrow$ not linear

$x^5-4 \,\big)\, 4x^8$

Oct 24-7:28 PM

Find the vertical, horizontal, and oblique asymptotes, if they exist, for the following functions.

5. $$f(x)=\frac{4x^5}{x^3-1} \quad n=5 \quad m=3$$

Use long division to find horizontal or oblique asymptotes:

no horizontal + no oblique

diff. of 2 cubes

$x^3-1 = (x-1)(x^2+x+1)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a=\sqrt[3]{x^3}=x$

$b=\sqrt[3]{1}=1$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

Vertical asymptote:

$x^3-1=0$

$x^3=1$

$x=1$

Nov 3-8:28 PM

Find the vertical, horizontal, and oblique asymptotes, if they exist, for the following functions.

6. $$g(x)=\frac{4x^4}{x^3-1} = (x-1)(x^2+x+1)$$

1) lowest terms

$n>m$

$n=m+1$

$$\begin{array}{r} 4x + \frac{4x}{x^3-1} \\ x^3-1 \,\big)\, 4x^4 \qquad \\ -4x^4 - 4x \\ \hline 0 + 4x \end{array}$$

oblique asym: $y=4x$

Vert. $x^3-1=0$

$x=1$

Nov 3-8:29 PM

Find the vertical, horizontal, and oblique asymptotes, if they exist, for the following functions.

7. $$h(x)=\frac{4x^3}{x^3-1} \quad n=m, \; y=\frac{4}{1}$$

horizontal asym: $y=4$

Vert. $x=1$

Nov 3-8:30 PM

Rate yourself on how well you understood this lesson.

I don't get it at all	I sort of get it	I understand most, but I need more practice	I understand it pretty well	I got it!!
1	2	3	4	5

What I still need to work on....

Aug 24-10:50 AM