Precalculus 5.7 Financial Models

Objective: able to determine the future value, present value, and rate of interest required to double a lump sum of money; calculate effective rates of return,

Simple Interest - If a principal of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, then the interest I charged is $I = P \cdot r t$

Compound interest is interest paid on previously earned interest.

1. Let P represent the principal to be invested at a per annum interest rate r that is to be compounded n times per year. Find the general formula for compound interest using a pattern of calculations.

$A = P + I$

$= P + Prt$ $\quad t = \frac{1}{n}$

$= P + Pr\left(\frac{1}{n}\right)$

$= P + P\left(\frac{r}{n}\right)$

$= P\left(1+\frac{r}{n}\right)$ new P

amount after 1 compounded period

after 2 compounded periods $\quad P\left(1+\frac{r}{n}\right) \cdot rt$

$A = P\left(1+\frac{r}{n}\right) + Prt$

$A = P\left(1+\frac{r}{n}\right) + P\left(1+\frac{r}{n}\right)\frac{r}{n}$

$= P\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)$

$= P\left(1+\frac{r}{n}\right)^2$

after 3 $\quad P\left(1+\frac{r}{n}\right)^3$

So..... $\quad P\left(1+\frac{r}{n}\right)^n$ stands for # of times compounded in a year

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Compound Interest Formula - The amount A after t years due to a principal P invested at an annual interest rate r compounded n times per years is $P\left(1+\frac{r}{n}\right)^{nt}$.

The amount A is typically referred to as the future value of the account, while P is called the present value.

2. Find the amount that results from investing $50 at 6% compounded monthly for 3 years.

$P = 50 \quad r = .06 \quad n = 12 \quad t = 3$

$50\left(1+\frac{.06}{12}\right)^{12(3)}$

$\approx \$59.83$

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Now, suppose that the number n of times that the interest is compounded per year gets larger and larger; that is suppose that $n \to \infty$. What will happen to the compound interest formula?

$A = P\left(1+\frac{r}{n}\right)^{nt}$ $\qquad \frac{1}{\frac{n}{r}} = \frac{n}{r}^{-1} = \frac{r}{n}$

$t = 1$ $\qquad$ let $\frac{n}{r} = h$

$= P\left(1+\frac{r}{n}\right)^n$

$= P\left(1+\frac{1}{\frac{n}{r}}\right)^n$ $\qquad P\left(\left(1+\frac{1}{h}\right)^h\right)^r$

$= P\left(\left(1+\frac{1}{\frac{n}{r}}\right)^{\frac{n}{r}}\right)^r$ $\qquad e = \left(1+\frac{1}{h}\right)^h$ as $h \to \infty$

Pe^r $\quad t = 1$

When # yrs. doesn't = 1 $\longrightarrow A = Pe^{rt}$

compounded continuously

Simple interest $\quad I = Prt$

Compounded interest when know n $\quad A = P\left(1+\frac{r}{n}\right)^{nt}$

Compounded continuously $\quad A = Pe^{rt}$

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3. Find the amount that results from investing $100 at 10% compounded continuously after 2 ¼ years.

$2.25 = t \quad P \quad r = .1$

$A = Pe^{rt}$

$A = 100e^{(.1)(2.25)}$

$\approx \$125.23$

4. Find the principal needed to get $100 after 2 years at 6% compounded monthly?

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$\frac{100}{\left(1+\frac{.06}{12}\right)^{24}} = \frac{P\left(1+\frac{.06}{12}\right)^{12 \cdot 2}}{\left(1+\frac{.06}{12}\right)^{24}}$ $\qquad A = P\left(1+\frac{r}{n}\right)^{nt}$

$\$88.72$

5. What annual rate of interest compounded annually is required to double an investment in 3 years?

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$t \qquad n = 1 \qquad 2P$

$\frac{2P}{P} = \frac{P\left(1+\frac{r}{1}\right)^{1(3)}}{P}$ $\qquad P > 0$

$\sqrt[3]{2} = \sqrt[3]{(1+r)^3}$

$\sqrt[3]{2} = 1 + r$ $\qquad r = .26$

$\sqrt[3]{2} - 1 = r$ $\qquad = 26\%$

6. How long will it take for an investment to triple in value if it is invested at 10% per annum compounded monthly? Compounded continuously?

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6. How long will it take for an investment to triple in value if it is invested at 10% per annum compounded monthly? Compounded continuously?

$n=12$ $3P$ $r=.1$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\frac{3\cancel{P}}{\cancel{P}} = \frac{\cancel{P}\left(1+\frac{.1}{12}\right)^{12t}}{\cancel{P}}$$

$P>0$

$$3 = \left(1+\frac{.1}{12}\right)^{12t}$$

$$\ln 3 = \ln\left(1+\frac{.1}{12}\right)^{12t}$$

$$\frac{\ln 3}{12\left(\ln\left(1+\frac{.1}{12}\right)\right)} = \frac{\cancel{12t}\cdot\left(\ln\left(1+\frac{.1}{12}\right)\right)}{\cancel{12}\left(\ln\left(1+\frac{.1}{12}\right)\right)}$$

≈ 11.032 yrs.

$$A = Pe^{rt}$$

$$3\cancel{P} = \cancel{P}e^{.1(t)}$$

$$\frac{\ln_e 3}{.1} = \frac{.1t}{.1}$$

≈ 10.986 yrs

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The **effective rate of interest** is the equivalent annual simple rate of interest that would yield the same amount as compounding after 1 year.

$$*r_e = \left(1+\frac{r}{n}\right)^n - 1 \qquad r_e = e^r - 1$$

7. Consider a principal of $1000 invested at an annual rate of 10% for one year. What is the effective rate of interest for compounding annually, quarterly, monthly, daily and continuously?

$n=1$	$n=4$	$n=12$	$n=365$	$n\to\infty$
$r_e = \left(1+\frac{.1}{1}\right)^1 - 1$	$=\left(1+\frac{.1}{4}\right)^4 - 1$	$=\left(1+\frac{.1}{12}\right)^{12} - 1$	$=\left(1+\frac{.1}{365}\right)^{365} - 1$	$e^{.1} - 1$
10%	$\approx 10.381\%$	$\approx 10.471\%$	$\approx 10.516\%$	$\approx 10.517\%$

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7. Consider a principal of $1000 invested at an annual rate of 10% for one year. What is the effective rate of interest for compounding annually, quarterly, monthly, daily and continuously?

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Rate yourself on how well you understood this lesson.

I don't get it at all	I sort of get it	I understand most, but I need more practice	I understand it pretty well	I got it!!
1	2	3	4	5

What I still need to work on....

Aug 24-10:50 AM