Assessment Title: Finding Forms of a Quadratic Summary
Unit 3: Quadratic Functions Working with Equations

Learning Targets:
- Use and interpret various forms of quadratics functions
- Identify key features of a quadratic function

As practiced in the lesson, if all key features (x-intercepts, y-intercept and vertex) are to be identified, it is sometimes necessary to express a function in an equivalent form. The forms and key features found include the following:

<table>
<thead>
<tr>
<th>Form</th>
<th>Key Features Easily Identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex Form: ( y = a(x - h)^2 + k )</td>
<td>vertex ((h, k))</td>
</tr>
<tr>
<td></td>
<td>y-intercept (replace (x) with 0, and simplify)</td>
</tr>
<tr>
<td>Factored Form: ( y = (x + a)(x + b) )</td>
<td>x-intercepts from factors</td>
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<tr>
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<td>y-intercept by replacing (x) with 0, and simplifying</td>
</tr>
<tr>
<td>Standard Form: ( y = ax^2 + bx + c )</td>
<td>y-intercept is (c) (replace (x) with 0, and simplify)</td>
</tr>
</tbody>
</table>

**PART 1: Vertex Form to Standard Form (Identifying the y-intercept):**

Use prior knowledge to simplify expression, and write terms in descending order.

Vertex Form

\[
y = 2(x - 1)^2 + 3
\]

\[
y = 2(x^2 - 2x + 1) + 3
\]

Multiply \((x - 1)(x - 1)\), and simplify.

\[
y = 2x^2 - 4x + 2 + 3
\]

Distribute 2.

Standard Form

\[
y = 2x^2 - 4x + 5
\]

Simplify.

TRY IT OUT: Write each quadratic function in standard form and identify the y-intercept of each function.

1. \( y = 3(x + 2)^2 + 1 \)

   Standard Form: _______________

   y-intercept: ( ________ , ________ )

2. \( y = (x - 2)^2 + 2 \)

   Standard Form: _______________

   y-intercept: ( ________ , ________ )

3. \( y = -(x + 1)^2 - 1 \)

   Standard Form: _______________

   y-intercept: ( ________ , ________ )

4. \( y = -2(x - 4)^2 - 3 \)

   Standard Form: _______________

   y-intercept: ( ________ , ________ )
PART 2: Factored Form to Standard Form:

Use prior knowledge to simplify expression, and write terms in descending order.

Factored Form

\[ y = 3(x - 2)(x - 1) \]
\[ y = 3(x^2 - x - 2x + 2) \]
\[ y = 3x^2 - 3x + 2 \]  Multiply \((x - 2)(x - 1)\) using Distributive Property.

Standard Form

\[ y = 3x^2 - 9x + 6 \]  Simplify.

TRY IT OUT: Write quadratic function in standard form.

1. \[ y = -2(x + 2)(x - 1) \]

2. \[ y = (x + 4)(x + 3) \]

y-intercept: ( ________, ________ )  y-intercept: ( ________, ________ )

3. \[ y = 3(2x - 1)(x - 1) \]

4. \[ y = (5x + 1)(x - 3) \]

y-intercept: ( ________, ________ )  y-intercept: ( ________, ________ )
PART 3: Standard Form to Factored Form (Identifying the x-intercepts):

This will require new knowledge in factoring quadratic expressions.

Standard Form \( y = 2x^2 + 4x - 6 \)
\( y = 2(x^2 + 2x - 3) \) Factor out common factor of 2.

Factored Form \( y = 2(x + 3)(x - 1) \) Factor quadratic expression.

TRY IT OUT: Write quadratic function in factored form.

1. \( y = x^2 + 5x + 6 \) \( x - \text{intercepts}: \)
2. \( y = 2x^2 - x - 1 \) \( x - \text{intercepts}: \)

3. \( y = x^2 - 5x + 6 \) \( x - \text{intercepts}: \)
4. \( y = x^2 - 5x - 6 \) \( x - \text{intercepts}: \)

5. \( y = 3x^2 - 6x + 3 \) \( x - \text{intercepts}: \)
6. \( y = x^2 - 3x - 10 \) \( x - \text{intercepts}: \)

7. \( y = x^2 - 4x + 4 \) \( x - \text{intercepts}: \)
8. \( y = 2x^2 + 4x - 70 \) \( x - \text{intercepts}: \)

9. \( y = x^2 - 17x + 72 \) \( x - \text{intercepts}: \)
10. \( y = 5x^2 + 35x + 60 \) \( x - \text{intercepts}: \)
PART 4: Standard Form to Vertex Form:

*Reason to change would be to identify vertex. This will require new knowledge in completing the square.

Standard Form

\[ y = x^2 + 4x - 6 \]
\[ y = (x^2 + 4x + 4) - 3 - 4 \]
Add 4 to complete the square. Subtract 4 to keep equation balanced.

Vertex Form

\[ y = (x + 2)^2 - 7 \]
Write in factored form, and simplify.

Standard Form

\[ y = 2x^2 + 4x - 10 \]
\[ y = 2(x^2 + 2x - 5) \]
Factor out 2.
\[ y = 2[(x^2 + 2x + 1) - 5 - 1] \]
Add 1 to complete the square. Subtract 1 to keep equation balanced.
\[ y = 2[(x + 1)^2 - 6] \]
Write in factored form, and simplify.

Vertex Form

\[ y = 2(x + 1)^2 - 12 \]
Distribute 2.

TRY IT OUT: Write quadratic function in vertex form.

1. \[ y = x^2 + 6x - 2 \]

   \[ \text{Vertex:} \]

2. \[ y = x^2 + 8x - 1 \]

   \[ \text{Vertex:} \]

3. \[ y = 2x^2 + 4x + 6 \]

   \[ \text{Vertex:} \]

4. \[ y = 3x^2 + 12x - 6 \]

   \[ \text{Vertex:} \]

5. \[ y = x^2 + 12x + 2 \]

   \[ \text{Vertex:} \]

6. \[ y = x^2 + 5x + 1 \]

   \[ \text{Vertex:} \]
ANSWER KEY

As practiced in the lesson, if all key features (x-intercepts, y-intercept and vertex) are to be identified, it is sometimes necessary to express a function in an equivalent form. The forms and key features found include the following:

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| Vertex Form: \( y = a(x - h)^2 + k \) | vertex \((h, k)\)  
 \( y \)-intercept (replace \(x\) with 0, and simplify) |
| Factored Form: \( y = (x + a)(x + b) \) | \( x \)-intercepts from factors  
 \( y \)-intercept by replacing \(x\) with 0, and simplifying |
| Standard Form: \( y = ax^2 + bx + c \) | \( y \)-intercept is \(c\) (replace \(x\) with 0, and simplify) |

PART 1: Vertex Form to Standard Form (Identifying the \(y\)-intercept):

Use prior knowledge to simplify expression, and write terms in descending order.

Vertex Form
\[
y = 2(x - 1)^2 + 3  
\]
\[
y = 2(x^2 - 2x + 1) + 3  
\]
Multiply \((x - 1)(x - 1)\), and simplify.
\[
y = 2x^2 - 4x + 2 + 3  
\]
Distribute 2.
Standard Form
\[
y = 2x^2 - 4x + 5  
\]
Simplify.

TRY IT OUT: Write each quadratic function in standard form and identify the \(y\)-intercept of each function.

1. \[
y = 3(x + 2)^2 + 1 \]
\[
= 3(x^2 + 4x + 4) + 1  
\]
\[
= 3x^2 + 12x + 12 + 1  
\]
Standard Form: \(3x^2 + 12x + 13\)
y-intercept: ( \(0\), \(13\) )

2. \[
y = (x - 2)^2 + 2 \]
\[
= (x - 2)(x - 2) + 2  
\]
\[
= x^2 - 4x + 4 + 2  
\]
\[
= x^2 - 4x + 6  
\]
Standard Form: \(x^2 - 4x + 6\)
y-intercept: ( \(0\), \(6\) )

3. \[
y = -(x + 1)^2 - 1 \]

4. \[
y = -2(x - 4)^2 - 3 \]
\[
= -2(x - 4)(x - 4) - 3  
\]
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\[ y = -\frac{(x+1)(x+1)}{1} \]
\[ = -\frac{(x^2 + 2x + 1)}{1} \]
\[ = -x^2 - 2x - 1 - 1 \]

**Standard Form:** \(x^2 - 2x - 2\)

y-intercept: \((0, -2)\)

\[ y = -2(x^2 - 8x + 16) - 3 \]
\[ = -2x^2 + 16x - 32 - 3 \]

**Standard Form:** \(-2x^2 + 16x - 35\)

y-intercept: \((0, -35)\)

**PART 2: Factored Form to Standard Form:**

Use prior knowledge to simplify expression, and write terms in descending order.

**Factored Form**
- \(y = 3(x - 2)(x - 1)\)
- \(y = 3(x^2 - x - 2x + 2)\)
- \(y = 3(x^2 - 3x + 2)\)

**Multiply** \((x - 2)(x - 1)\) using Distributive Property.

**Simplify.**

**Standard Form**
- \(y = 3x^2 - 9x + 6\)

Distribute 3.

TRY IT OUT: Write quadratic function in standard form.

1. \(y = -2(x + 2)(x - 1)\)
\[ = -2(x^2 + x - 2) \]
\[ = -2x^2 - 2x + 4 \]

y-intercept: \((0, 4)\)

2. \(y = (x + 4)(x + 3)\)
\[ = x^2 + 7x + 12 \]

y-intercept: \((0, 12)\)

3. \(y = 3(2x - 1)(x - 1)\)
\[ = 3(2x^2 - 3x + 1) \]
\[ = 6x^2 - 9x + 3 \]

y-intercept: \((0, 3)\)

4. \(y = (5x + 1)(x - 3)\)
\[ = 5x^2 - 14x - 3 \]

y-intercept: \((0, -3)\)
PART 3: Standard Form to Factored Form (Identifying the x-intercepts):

This will require new knowledge in factoring quadratic expressions.

Standard Form

\[ y = 2x^2 + 4x - 6 \]

\[ y = 2(x^2 + 2x - 3) \]

Factored Form

\[ y = 2(x + 3)(x - 1) \]

Factor out common factor of 2.

Factor quadratic expression.

TRY IT OUT: Write quadratic function in factored form.

1. \[ y = x^2 + 5x + 6 \]

\[ = (x+3)(x+2) \]

\[ x - \text{intercepts}: (-3,0) \& (-2,0) \]

2. \[ y = 2x^2 - x - 1 \]

\[ = (-2x-1)(x+1) \]

\[ x - \text{intercepts}: (-\frac{1}{2},0) \& (1,0) \]

3. \[ y = x^2 - 5x + 6 \]

\[ = (x-3)(x-2) \]

\[ x - \text{intercepts}: (3,0) \& (2,0) \]

4. \[ y = x^2 - 5x - 6 \]

\[ = (x-6)(x+1) \]

\[ x - \text{intercepts}: (6,0) \& (-1,0) \]

5. \[ y = 3x^2 - 6x + 3 \]

\[ = (3x-3)(x-1) \]

\[ x - \text{intercepts}: (1,0) \]

6. \[ y = x^2 - 3x - 10 \]

\[ = (x-5)(x+2) \]

\[ x - \text{intercepts}: (5,0) \& (-2,0) \]

7. \[ y = x^2 - 4x + 4 \]

\[ = (x-2)(x-2) \]

\[ x - \text{intercepts}: (2,0) \]

8. \[ y = 2x^2 + 4x - 70 \]

\[ = (2x-10)(x+7) \]

\[ x - \text{intercepts}: (5,0) \& (-7,0) \]

9. \[ y = x^2 - 17x + 72 \]

\[ = (x-8)(x-9) \]

\[ x - \text{intercepts}: (8,0) \& (9,0) \]

10. \[ y = 5x^2 + 35x + 60 \]

\[ = (5x+20)(x+3) \]

\[ x - \text{intercepts}: (5,0) \& (-3,0) \]
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PART 4: Standard Form to Vertex Form:

*Reason to change would be to identify vertex. This will require new knowledge in completing the square.

Standard Form

\[
\begin{align*}
\text{y} &= x^2 + 4x - 6 \\
\text{y} &= (x^2 + 4x + 4) - 3 - 4
\end{align*}
\]

Add 4 to complete the square. Subtract 4 to keep equation balanced.

Vertex Form

\[
\begin{align*}
\text{y} &= (x + 2)^2 - 7
\end{align*}
\]

Write in factored form, and simplify.

Standard Form

\[
\begin{align*}
\text{y} &= 2x^2 + 4x - 10 \\
\text{y} &= 2(x^2 + 2x - 5) \\
\text{y} &= 2[(x^2 + 2x + 1) - 5 - 1]
\end{align*}
\]

Factor out 2. Add 1 to complete the square. Subtract 1 to keep equation balanced.

Vertex Form

\[
\begin{align*}
\text{y} &= 2[(x + 1)^2 - 6] \\
\text{y} &= 2(x + 1)^2 - 12
\end{align*}
\]

Write in factored form, and simplify.

TRY IT OUT: Write quadratic function in vertex form.

1. \( y = x^2 + 6x - 2 \)
\[
\begin{align*}
= (x^2 + 6x + 9) - 2 - 9 \\
= (x + 3)^2 - 11
\end{align*}
\]

Vertex: \((-3, -11)\)

2. \( y = x^2 + 8x - 1 \)
\[
\begin{align*}
= (x^2 + 8x + 16) - 1 - 16 \\
= (x + 4)^2 - 17
\end{align*}
\]

Vertex: \((-4, -17)\)
Math 2 (L1-6)

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3. \( y = 2x^2 + 4x + 6 \)
   \[
   = \frac{2(x^2 + 2x + 3)}{2} + 3 - 1
   = 2(x + 1)^2 + 2
   \]
   Vertex: \((-1, 2)\)

4. \( y = 3x^2 + 12x - 6 \)
   \[
   = \frac{3(x^2 + 4x - 2)}{3} - 2 - 4
   = 3(x + 2)^2 - 6
   \]
   Vertex: \((-2, -6)\)

5. \( y = x^2 + 12x + 2 \)
   \[
   = \frac{(x^2 + 12x + 36) + 2 - 36}{(x + 6)^2 - 34}
   \]
   Vertex: \((-6, -34)\)

6. \( y = x^2 + 5x + 1 \)
   \[
   = \frac{(x^2 + 5x + \frac{25}{4}) + 1 - \frac{25}{4}}{(x + \frac{5}{2})^2 - \frac{21}{4}}
   \]
   Vertex: \((\frac{5}{2}, -\frac{21}{4})\)